Waveguide structures

- The basic structure of a dielectric waveguide consists of a longitudinally extended high-index optical medium, called the core, which is transversely surrounded by low-index media, called the cladding. A guided optical wave propagates in the waveguide along its longitudinal direction.

- The characteristics of a waveguide are determined by the transverse profile of its dielectric constant \( \varepsilon(x, y)/\varepsilon_0 \), which is independent of the longitudinal \( z \) direction.

- For a waveguide made of optically isotropic media, we can characterize the waveguide with a single spatially dependent transverse profile of the index of refraction \( n(x, y) \).

Nonplanar and planar waveguides

- There are two basic types of waveguides:
  - In a nonplanar waveguide of two-dimensional transverse optical confinement, the core is surrounded by cladding in all transverse directions, and \( n(x, y) \) is a function of both \( x \) and \( y \) coordinates. E.g. the channel waveguides and the optical fibers.

- In a planar waveguide that has optical confinement in only one transverse direction, the core is sandwiched between cladding layers in only one direction, say the \( x \) direction, with an index profile \( n(x) \). The core of a planar waveguide is also called the film, while the upper and lower cladding layers are called the cover and the substrate.

Optical waveguides

- Optical waveguides are the basic elements for confinement and transmission of light over various distances, ranging from tens or hundreds of \( \mu \)m in integrated photonics to hundreds or thousands of km in long-distance fiber-optic transmission. Optical waveguides also form key structures in semiconductor lasers, and act as passive and active devices such as waveguide couplers and modulators.
Index profiles

- A waveguide in which the index profile has abrupt changes between the core and the cladding is called a **step-index waveguide**, while one in which the index profile varies gradually is called a **graded-index waveguide**.
- We will focus on the step-index waveguide for our discussion.

![Step-index waveguide diagram](image)

Channel waveguides

- Most waveguides used in device applications are **nonplanar waveguides**.
- For a nonplanar waveguide, the index profile \( n(x, y) \) is a function of both transverse coordinates \( x \) and \( y \).
- There are many different types of nonplanar waveguides that are differentiated by the distinctive features of their index profiles.
- One very unique group is the **circular optical fibers** (to be discussed in Lecture 5).
- Another important group of nonplanar waveguides is the **channel waveguides**, which include:
  - The buried channel waveguides
  - The strip-loaded waveguides
  - The ridge waveguides
  - The rib waveguides
  - The diffused waveguides.

Representative channel waveguides

- A **buried channel waveguide** is formed with a high-index waveguiding core buried in a low-index surrounding medium. The waveguiding core can have any cross-sectional geometry though it is often a rectangular shape.
- A **strip-loaded waveguide** is formed by loading a planar waveguide, which already provides optical confinement in the \( x \) direction, with a dielectric strip of index \( n_3 < n_1 \) or a metal strip to facilitate optical confinement in the \( y \) direction. The waveguiding core of a strip waveguide is the \( n_1 \) region under the loading strip, with its thickness \( d \) determined by the thickness of the \( n_1 \) layer and its width \( w \) defined by the width of the loading strip.
- A **ridge waveguide** has a structure that looks like a strip waveguide, but the strip, or the ridge, on top of its planar structure has a high index and is actually the waveguiding core. A ridge waveguide has strong optical confinement because it is surrounded on three sides by low-index air (or cladding material).
Representative channel waveguides

- A **rib waveguide** has a structure similar to that of a strip or ridge waveguide, but the strip has the same index as the high-index planar layer beneath it and is part of the waveguiding core.
- These four types of waveguides are usually termed **rectangular waveguides** with a thickness $d$ in the $x$ direction and a width $w$ in the $y$ direction, though their shapes are normally not exactly rectangular.
- A **diffused waveguide** is formed by creating a high-index region in a substrate through diffusion of dopants, such as LiNbO$_3$ waveguide with a core formed by Ti diffusion.
- Because of the diffusion process, the core boundaries in the substrate are not sharply defined.
- A diffused waveguide also has a thickness $d$ defined by the diffusion depth of the dopant in the $x$ direction and a width $w$ defined by the distribution of the dopant in the $y$ direction.

Remarks on nonplanar waveguides

- One distinctive property of **nonplanar** dielectric waveguides versus planar waveguides is that a **nonplanar waveguide supports hybrid modes** in addition to TE and TM modes, whereas a planar waveguide supports only TE and TM modes.
- Except for those few exhibiting special geometric structures, such as circular optical fibers, **non-planar** dielectric waveguides generally do not have analytical solutions for their guided mode characteristics.
- Numerical methods, such as the **beam propagation method**, exist for analyzing such waveguides. Here we discuss approximate solutions that give the mode characteristics. One of the methods is the **effective index method**.

Silicon optical waveguides (nanophotonic wires)

- Guide light by total internal reflection in a few 100 nm cross-section (propagation loss typically few - ~1 dB/cm)

Solving waveguide modes by numerical methods

- Except for those few exhibiting special geometric structures, such as circular optical fibers, **non-planar** dielectric waveguides generally do not have analytical solutions for their guided mode characteristics.
- **Numerical methods**, such as the **beam propagation method**, are typically used for analyzing such waveguides (e.g. silicon-on-insulator waveguides modes, TE and TM mode electric field distributions)
Waveguide modes exist that are characteristic of a particular waveguide structure. A waveguide mode is a transverse field pattern whose amplitude and polarization profiles remain constant along the longitudinal $z$ coordinate. Therefore, the electric and magnetic fields of a mode can be written as follows

$$E_\nu(r,t) = E_\nu(x,y) \exp(i \beta_\nu z - \omega t)$$

$$H_\nu(r,t) = H_\nu(x,y) \exp(i \beta_\nu z - \omega t)$$

where $\nu$ is the mode index, $E_\nu(x,y)$ and $H_\nu(x,y)$ are the mode field profiles, and $\beta_\nu$ is the propagation constant of the mode.

For a waveguide of two-dimensional transverse optical confinement, there are two degrees of freedom in the transverse $xy$ plane, and the mode index $\nu$ consists of two parameters for characterizing the variations of the mode fields in these two transverse dimensions. E.g., $\nu$ represents two mode numbers, $\nu = mn$ with integral $m$ and $n$, for discrete guided modes.

For the planar waveguide, the mode fields do not depend on the $y$ coordinate. Thus, the electric and magnetic fields of a mode can be simplified to

$$E_\nu(r,t) = E_\nu(x) \exp(i \beta_\nu z - \omega t)$$

$$H_\nu(r,t) = H_\nu(x) \exp(i \beta_\nu z - \omega t)$$

In this case, $\nu$ consists of only one parameter characterizing the field variation in the $x$ dimension.

Consider the qualitative behavior of an optical wave in an asymmetric planar step-index waveguide, where $n_1 > n_2 > n_3$. For an optical wave of angular frequency $\omega$ and free-space wavelength $\lambda_0$, the media in the three different regions of the waveguide define the following propagation constants:

$$k_1 = n_1 \omega / c, k_2 = n_2 \omega / c, k_3 = n_3 \omega / c$$

where $k_1 > k_2 > k_3$.

We can obtain useful intuitive picture from considering the path of an optical ray, or a plane optical wave, in the waveguide.
There are twocritical anglesassociated with the internal reflections at the lower and upper interfaces:
\[ \theta_{c2} = \sin^{-1} \frac{n_2}{n_1}, \quad \theta_{c3} = \sin^{-1} \frac{n_3}{n_1} \]
\[ \theta_{c2} > \theta_{c3} \text{ because } n_2 > n_3 \]

If \( \theta > \theta_{c2} > \theta_{c3} \), the wave inside the core is totally reflected at both interfaces and is trapped by the core, resulting in guided modes.

**k-vector triangle**

- The orthogonal components of the propagation constant, \( \beta \) and \( k_x \), are related by the “k-vector triangle.”

\[ \begin{align*}
\text{core } n_1 & \quad \beta \\
& \quad k_x
\end{align*} \]

- Transverse component: \( k_x = (n_1\omega/c) \cos \theta \)
- Longitudinal component: \( \beta = (n_1\omega/c) \sin \theta \)
- “k-vector triangle”: \( \beta^2 + k_x^2 = (n_1\omega/c)^2 \)

**Guided modes**

- As the wave is reflected back and forth between the two interfaces, it interferes with itself.
- A guided mode can exist only when a transverse resonance condition is satisfied s.t. the repeatedly reflected wave has constructive interference with itself.
- In the core region, the x component of the wavevector is \( k_x = k_1 \cos \theta \) for a ray with an angle of incidence \( \theta \), while the z component is \( \beta = k_1 \sin \theta \).
- The phase shift in the optical field due to a round-trip transverse passage in the core of thickness \( d \) is \( 2k_1d \cos \theta \).

**k\( _x \) and \( \beta \) components**

- We can consider the “zig-zag” wave in the waveguide as two orthogonal components traveling in the longitudinal (z) and transverse (x) directions.
- The transverse component of the plane wave is reflected back and forth in the x direction, interfering with itself.
Transverse resonance condition

- There are phase shifts $\varphi_2$ and $\varphi_3$ associated with the internal reflections in the lower and upper interfaces.
- Recall from Lecture 1 that these phase shifts can be obtained from the phase angle of $r_s$ for a TE wave (s wave) and that of $r_p$ for a TM wave (p wave) for a given $\theta > \theta_{c2}, \theta_{c3}$.
- Because $\varphi_2$ and $\varphi_3$ are functions of $\theta$, the transverse resonance condition for constructive interference in a round-trip transverse passage is

$$2k_0d \cos \theta + \varphi_2(\theta) + \varphi_3(\theta) = 2m\pi$$

where $m$ is an integer $= 0, 1, 2, \ldots$
- Because $m$ can assume only integral values, only certain discrete values of $\theta$ can satisfy the transverse resonance condition.

Transverse electric polarization

- For slab waveguides, we define the x-z plane as the plane of incidence.
- An electric field pointing in the y direction corresponds to the perpendicular, or s, polarization.
- Waves with this polarization are labeled transverse electric (TE) fields because the electric field vector lies entirely in the x y plane (i.e. $E_z = 0$) that is transverse to the direction of net travel (the z direction).

Discrete guided modes

- The transverse resonance condition results in discrete values of the propagation constant $\beta_m$ for guided modes identified by the mode number $m$.
- Although the critical angles, $\theta_{c2}$ and $\theta_{c3}$, do not depend on the polarization of the wave, the phase shifts, $\varphi_2(\theta)$ and $\varphi_3(\theta)$, caused by the internal reflection at a given angle $\theta$ depend on the polarization.
- Therefore, TE and TM waves have different solutions for the transverse resonance condition, resulting in different $\beta_m$ and different mode characteristics for a given mode number $m$.
- For a given polarization, solution of the transverse resonance condition yields a smaller value of $\theta$ and a correspondingly smaller value of $\beta$ for a larger value of $m$. Therefore, $\beta_0 > \beta_1 > \beta_2 > \ldots$
- The guided mode with $m = 0$ is called the fundamental mode and those with $m \neq 0$ are higher-order modes.
Qualitative picture of a waveguide mode

The stable field distribution in the transverse direction with only a periodic longitudinal dependence is known as a waveguide mode.

Plane wave representation for a planar waveguide

Upward field: \( E_u = E_0 \exp(i k_u \cdot r) = E_0 \exp(i h x) \exp(i \beta z) \)

Downward field: \( E_d = E_0 \exp(i k_d \cdot r) = E_0 \exp(-i h x) \exp(i \beta z) \)

where \( r = x a_x + z a_z \); \( k_u = h a_x + \beta a_z \), \( k_d = -h a_x + \beta a_z \)

Discrete waveguide modes

- Because \( m \) can assume only integral values, only certain discrete values of \( \theta = \theta_m \) can satisfy the resonance condition.

- This results in discrete values of the propagation constant \( \beta_m \) for guided modes identified by the mode number \( m \).

- The guided mode with \( m = 0 \) is called the fundamental mode and those with \( m = 1, 2, \ldots \) are higher-order modes.
Surface waves and the reflective phase shift

• An electric field component in the upper cladding region assumes the phasor form:

\[ E_3 = E_{30} \exp(i k_3 \cdot r) = E_{30} \exp(i k_{3x} x) \exp(i \beta z) \]

• When \( \theta_1 > \theta_c3 \), \( k_{3x} \) becomes imaginary and can be expressed in terms of a real attenuation coefficient \( \kappa \) as

\[ k_{3x} = i \kappa \quad (\theta_1 > \theta_c3) \]

\[ E_3 = E_{30} \exp(-\kappa x) \exp(i \beta z) \]

• This is the phasor expression for a surface or evanescent wave – propagates only in the \( z \) direction, decays in the direction normal to the interface.

Evanescent field in total internal reflection

• Phase-matching at TIR \( \theta_1 > \theta_c \) (i.e. \( \sin \theta_1 > n_2/n_1 \))

\[ \beta_1 = n_1 k \sin \theta_1 = \beta_2 > n_2 k \]

• \( k \)-vector triangle in \( n_2 \)

\[ k_{2x} = [(n_2 k)^2 - (n_1 k \sin \theta_1)^2]^{1/2} \]

\[ k_{2x} = i[(n_1 k \sin \theta_1)^2 - (n_2 k)^2]^{1/2} \]

\[ = i k [(n_1 \sin \theta_1)^2 - n_2^2]^{1/2} = i \kappa \]

• evanescent field in the transverse direction

\[ E \propto \exp(-\kappa x) \exp(i \beta z - \omega t) \]

Phase-matching at an interface

• As the spatial rate of change of phase at the boundary (or the projection of the wavefront propagation) on the \( n_1 \) side must match with that on the \( n_2 \) side, we have \( \beta_1 = \beta_2 = \beta \). This condition is known as “phase-matching.” Phase-matching allows coupling of oscillating field between two media.

\[ \beta_1 = \beta_2 \Rightarrow n_1 k \sin \theta_1 = n_2 k \sin \theta_2 \quad (\text{Snell’s Law}) \]

Evanescent fields in the waveguide cladding

• Evanescent fields outside the waveguide core decay exponentially with an attenuation factor given by

\[ \kappa = k (n_1^2 \sin^2 \theta_1 - n_2^2) \]

• In the upper layer (\( x \geq d/2 \))

\[ E = E_2 \exp(-\kappa (x - d/2)) \exp(i(\beta z - \omega t)) \]

• In the lower layer (\( x \leq -d/2 \))

\[ E = E_2 \exp(\kappa (x + d/2)) \exp(i(\beta z - \omega t)) \]

where \( E_2 \) is the peak value of the electric field at the lower (\( x = -d/2 \)) and upper (\( x = d/2 \)) boundaries.
Waveguide effective index

- We can define the waveguide phase velocity \( v_p \) as
  \[
  v_p = \frac{\omega}{\beta}
  \]

- We now define an effective refractive index \( n_{\text{eff}} \) as the free-space velocity divided by the waveguide phase velocity.
  \[
  n_{\text{eff}} = \frac{c}{v_p}
  \]
  Or \( n_{\text{eff}} = \frac{c\beta}{\omega} = \frac{\beta}{k} \)
  \[
  \Rightarrow n_{\text{eff}} = n_1 \sin \theta
  \]

- The effective refractive index is a key parameter in guided propagation, just as the refractive index is in unguided wave travel.

- For waveguiding at \( n_1 \)-\( n_2 \) interface, we see that \( n_2 \leq n_{\text{eff}} \leq n_1 \)
  At \( \theta = 90^\circ \), \( n_{\text{eff}} = n_1 \Rightarrow \) A ray traveling parallel to the slab (core) has an effective index that depends on the guiding medium alone.
  At \( \theta = \theta_c \), \( n_{\text{eff}} = n_2 \Rightarrow \) The effective index for critical-angle rays depends only on the outer material \( n_2 \).

- The effective refractive index changes with the wavelength (i.e. dispersion) in a way related to that the bulk refractive index does.

- The wavelength as measured in the waveguide is
  \[
  \lambda_z = \frac{\lambda}{n_{\text{eff}}}
  \]

Field equations

- For a linear, isotropic dielectric waveguide characterized by a spatial permittivity distribution \( \varepsilon(x, y) \), Maxwell’s equations can be written as
  \[
  \nabla \times E = -\mu_0 \frac{\partial H}{\partial t}
  \]
  \[
  \nabla \times H = \varepsilon \frac{\partial E}{\partial t}
  \]

- Because the optical fields in the waveguide have the form of
  \[
  E_\varphi(r, t) = E_\varphi(x, y) \exp i(\beta z - \omega t)
  \]
  \[
  H_\varphi(r, t) = H_\varphi(x, y) \exp i(\beta z - \omega t)
  \]
Field equations

These two Maxwell's equations can be written as:

\[
\begin{align*}
\frac{\partial E_x}{\partial y} - i\beta E_y &= i\omega \mu_0 H_z, \\
\frac{\partial E_y}{\partial x} - i\beta E_x &= i\omega \mu_0 H_z, \\
\frac{\partial H_z}{\partial x} &= -i\omega \varepsilon E_y, \\
\frac{\partial H_z}{\partial y} &= -i\omega \varepsilon E_x.
\end{align*}
\]

The transverse components of the electric and magnetic fields can be expressed in terms of the longitudinal components:

\[
\begin{align*}
(k^2 - \beta^2) E_x &= i\beta \frac{\partial E_x}{\partial x} + i\omega \mu_0 \frac{\partial H_z}{\partial y} \\
(k^2 - \beta^2) H_x &= i\beta \frac{\partial H_x}{\partial x} - i\omega \varepsilon \frac{\partial E_y}{\partial y} \\
(k^2 - \beta^2) E_y &= i\beta \frac{\partial E_y}{\partial y} - i\omega \mu_0 \frac{\partial H_z}{\partial x} \\
(k^2 - \beta^2) H_y &= i\beta \frac{\partial H_y}{\partial y} + i\omega \varepsilon \frac{\partial E_x}{\partial x}.
\end{align*}
\]

The relations on the transverse components of the \( E \) and \( H \) fields are generally true for a longitudinally homogeneous waveguide of any transverse geometry and any transverse index profile where \( \varepsilon(x, y) \) is not a function of \( z \). They are equally true for step-index and graded-index waveguides.

In waveguides that have circular cross sections, such as optical fibers, the \( x \) and \( y \) coordinates of the rectangular system can be transformed to the \( r \) and \( \phi \) coordinates of the cylindrical system for similar relations.

Polarization modes

The fields in a waveguide can have various vectorial characteristics.

They can be classified based on the characteristics of the longitudinal field components:

- A transverse electric and magnetic mode, or TEM mode, has \( E_z = 0 \) and \( H_z = 0 \). Dielectric waveguides do not support TEM modes.
- A transverse electric mode, or TE mode, has \( E_z = 0 \) and \( H_z \neq 0 \).
- A transverse magnetic mode, or TM mode, has \( H_z = 0 \) and \( E_z \neq 0 \).
- A hybrid mode has both \( E_z \neq 0 \) and \( H_z \neq 0 \). Hybrid modes do not appear in planar waveguides but exist in nonplanar waveguides of two-dimensional transverse optical confinement. E.g., We will see that the HE and EH modes of optical fibers are hybrid modes.
Wave equations

- The common approach to finding $E_z$ and $H_z$ is to solve the wave equations together with boundary conditions.
- For the case of a linear, isotropic waveguide with a spatially dependent $\varepsilon(x, y)$, the two Gauss’ laws for $E$ and $H$ can be written as
  \[ \nabla \cdot (\varepsilon E) = 0 \Rightarrow \nabla \cdot E = -\frac{\nabla \varepsilon}{\varepsilon} \cdot E \]
  \[ \nabla \cdot H = 0 \]
- Note that $\nabla \cdot E$ does not vanish in general because $\varepsilon(x, y)$ is spatially dependent.
- Using the four Maxwell’s equations, we have
  \[ \nabla^2 E + k^2 E = -\nabla \left( \frac{\nabla \varepsilon}{\varepsilon} \cdot E \right) \]
  \[ \nabla^2 H + k^2 H = -\frac{\nabla \varepsilon}{\varepsilon} \times \nabla \times H \]

Wave equations for step-index waveguides

- The index profile of a step-index waveguide is piecewise constant.
- We can write a homogeneous wave equation separately for each region of constant $\varepsilon$ because $\nabla \varepsilon = 0$ within each region.
- Assuming $E$ and $H$ of the harmonic guided wave form, and with $\nabla \varepsilon = 0$ for each region of constant $\varepsilon$, we obtain for the longitudinal components
  \[ \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (k_i^2 - \beta^2) E_z = 0 \]
  \[ \frac{\partial^3 H_z}{\partial x^2} + \frac{\partial^3 H_z}{\partial y^2} + (k_i^2 - \beta^2) H_z = 0 \]
  where $k_i^2 = \omega^2 \mu_0 \varepsilon_i = n_i^2 \omega^2 / c^2$ is a constant for region $i$, which has a constant index of refraction $n_i$. 

Wave equations

- The three components $E_x, E_y$ and $E_z$ for the electric field are generally coupled together because $\nabla \varepsilon \neq 0$ in a waveguide. For the same reason $H_x, H_y$ and $H_z$ are also coupled.
- This fact indicates that the vectorial characteristics of a mode field in a waveguide are strongly dependent on the geometry and index profile of the waveguide.
- In the case of a TE mode, $\nabla \varepsilon \perp E$ s.t. $\nabla \varepsilon \cdot E = 0$
- Thus, each component of the electric field of a TE mode satisfies a homogeneous scalar differential equation. The magnetic field components of a TE mode are still coupled.

Wave equations for step-index waveguides

- A homogeneous equation in the same form can be written for each of the other four field components, $E_x, E_y, H_x$ and $H_y$.
- However, it is not necessary to solve the wave equations for all field components because the transverse field components can be found from $E_z$ and $H_z$.
- Therefore, the mode field pattern can be obtained by solving the two equations above for each region of constant index and by requiring the fields to satisfy the boundary conditions at the interfaces between neighboring regions.
- However, note that this approach does not work for graded-index waveguides because $\nabla \varepsilon \neq 0$ for such waveguides.
Wave equations for planar waveguides

- Homogeneous wave equations exist for planar waveguides of any index profile \( n(x) \).
- For a planar waveguide, the modes are either TE or TM.
- Furthermore, we consider \( \partial/\partial y = 0 \) because the index profile is independent of the \( y \) coordinate. The wave equations are substantially simplified.

For any TE mode of a planar waveguide, \( E_z = 0 \).

Then, \( H_x = E_y = 0 \) because \( \partial E_y / \partial y = 0 \).

The only nonvanishing field components are \( H_x, \ E_y \) and \( H_z \).

Because there is only one nonvanishing electric field component \( E_y \), the wave equation for \( E_y \) is naturally decoupled from the other field components. Therefore, we have

\[
\frac{\partial^2 E_y}{\partial x^2} + (k^2 - \beta^2)E_y = 0
\]

where \( k^2 = \omega^2 \mu_0 \varepsilon(x) = (\omega^2/c^2)n^2(x) \).

\( H_x \) and \( H_z \) can be obtained from \( E_y \) using the field equations:

\[
H_x = -\frac{\beta}{\omega \mu_0} E_y \quad \quad H_z = \frac{1}{i \omega \mu_0} \frac{\partial E_y}{\partial x}
\]

Guided modes in symmetric slab waveguides

- For any TM mode of a planar waveguide, \( E_z = 0 \).
- Such that from relations of the transverse components to the longitudinal components, we see that \( E_x = H_y = 0 \) because \( \partial H_y / \partial y = 0 \).
- The only nonvanishing field components are \( H_x, \ E_x \) and \( E_z \).
- The wave equation for \( H_y \)

\[
\frac{\partial^2 H_y}{\partial x^2} + (k^2 - \beta^2)H_y = \frac{1}{\varepsilon} \frac{d\varepsilon}{dx} \frac{\partial H_y}{\partial x}
\]

where \( k^2 = k^2(x) = (\omega^2/c^2)n^2(x) \).

\( E_x \) and \( E_z \) can be obtained from \( H_y \):

\[
E_x = \frac{\beta}{\omega \varepsilon} H_y \quad \quad E_z = -\frac{1}{i \omega \varepsilon} \frac{\partial H_y}{\partial x}
\]